Hands-On Method for Teaching the Concept of the Ratio between Surface Area & Volume
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Hands-On Method for Teaching the Concept of the Ratio Between Surface Area & Volume

Annette Cohen Anat Ben Moreh Reuben Chayoth

The concept of the ratio between surface area and volume appears in many biological systems. Teachers often refer to this ratio, without really knowing if the students fully comprehend this concept (Gilbert 1982). Research has shown that high school students who are faced with systems based upon this concept have difficulties in understanding that changing the shape and size of an object causes changes in the object's surface area, volume, and the relationship between surface area and volume (Cohen 1994; Livne 1996).

Possible explanations for students' difficulty in understanding this are as follows:

1. **Difficulty in understanding the independent variables *surface area* and *volume*.** Enochs and Gabel (1984) showed that these concepts are not fully understood even by elementary school teachers. They found that teachers tried to solve problems involving these variables by simply using mathematical equations from memory, without actually understanding the concepts. As a result, their teaching was based on rote calculations and not on understanding.
2. **Difficulty in isolating the variables.** When considering the ratio surface area /volume it is necessary to understand that while both surface area and volume relate to the same object, each of these variables can change independently of the other. Students find this very confusing and teachers have difficulty demonstrating this point to them in a concrete manner (Stavy & Tirosh 1996).
3. **Difficulty in understanding the abstract concept of *ratio*.** Students often solve mathematical problems involving ratios by learning techniques, while they do not actually understand the abstract meaning of *ratio* (Heller 1989). In addition, comprehension of this concept is characterized by formal (abstract) thinking, which many students have not as yet acquired.

There are several instances where the ratio *surface area to volume* varies, and we will examine four of these cases.

A. Relationship between surface area and volume in flattened objects.

When comparing objects of the same volume, it can be seen that the flatter the object the greater the proportion between surface area and volume. This principle can be demonstrated by comparing box shapes (Table 1). From the above table it can be seen that the flatter a body, the larger the surface area in comparison to weight or body volume. The advantages of large surface area/volume in living organisms are many. The flatter the organism or the individual cell, the more efficient the exchange of substances with the environment. This principle can be clearly seen in the following examples:

1. Red blood cells have a unique flattened shape. The depression on each flat surface of the cell results in a thin center and thicker edges. This shape gives the red blood cell a very large surface area relative to its volume (Thibodeau & Patton 1993a).
2. Elephants have large flat ears which enable them to cool their body by evaporation of water.
3. Flatworms, which are mostly parasites living in animal body fluids, receive their nutrition by diffusion. In this case the advantages to being flat are obvious, and the short distance of diffusion to all cells enables more efficient feeding.

B. Relationship between surface area and volume in elongated objects.

As similar to the case where objects become flatter, when objects become

Table 1. Relationship between surface area and volume in flattened objects.

Box No.	*Length (cm)*	*Width (cm)*	*Height (cm)*	*Surface Area (cm^2)*	*Volume (cm^3)*	*Surface Area/Vol.*
1	8	8	8	384	512	0.75
2	16	8	4	448	512	0.87
3	16	16	2	640	512	1.25
4	32	16	1	1120	512	2.19
5	32	32	0.5	2112	512	4.125

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Table 2. Relationship between surface area and volume in elongated objects.

Box No.	*Length (cm)*	*Width (cm)*	*Height (cm)*	*Surface Area (cm^2)*	*Volume (cm^3)*	*Surface Area/Vol.*
1	8	8	8	384	512	0.75
2	16	8	4	448	512	0.87
3	32	4	4	544	512	1.06
4	64	4	2	784	512	1.53
5	128	2	2	1032	512	2.02

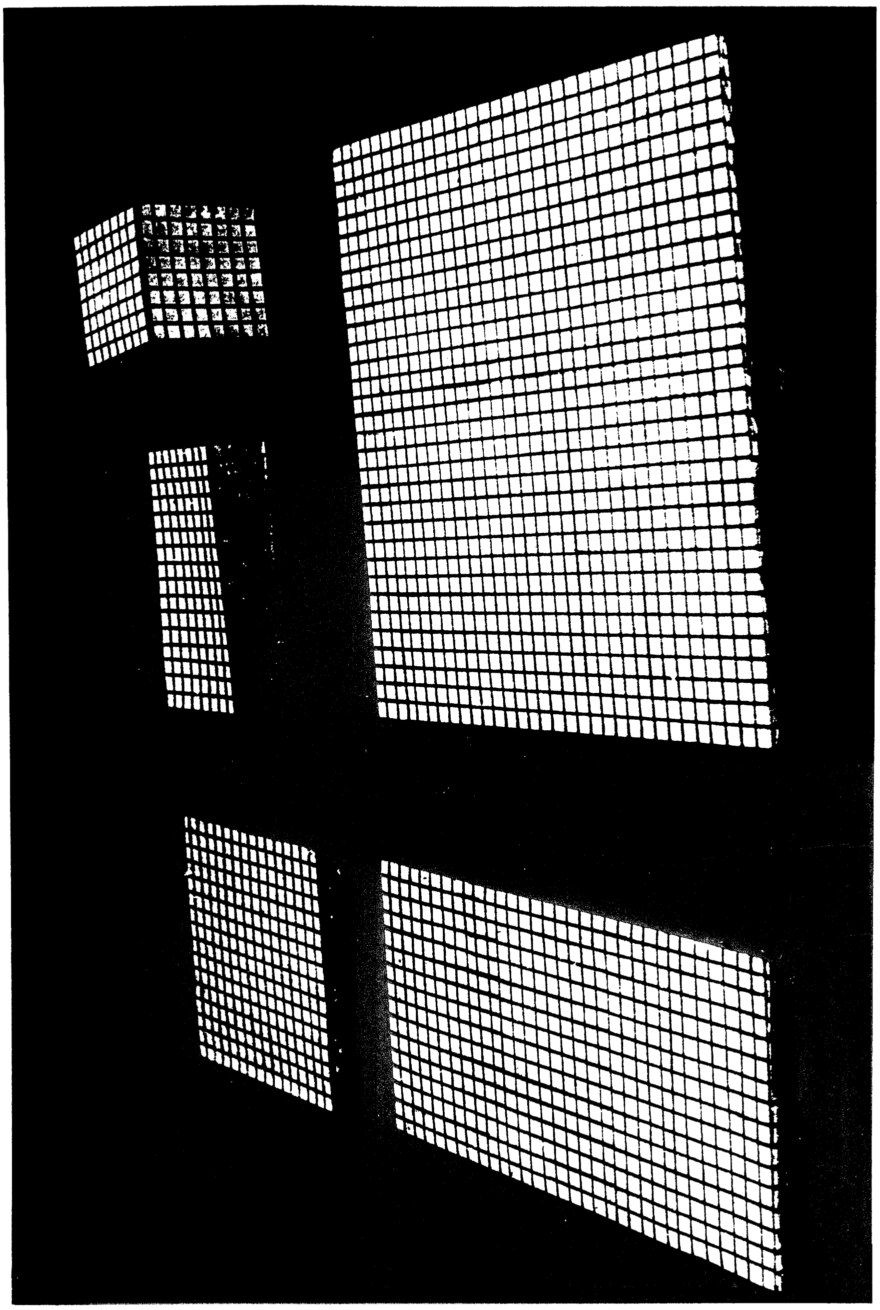

Figure 1. Models used to demonstrate the relationship between surface area and volume in flattened objects. The models go from a square box to more and more flattened shapes. (Each grid square represents one square cm.)

longer without changing their volume, the ratio between surface area to volume increases. This relationship is demonstrated in Table 2.

This phenomenon of elongation accompanied by increase in surface area in relationship to volume is often found in biological systems. A few examples follow:

1. Capillary blood vessels, through which substances are exchanged between blood and intercellular fluids, are very thin structures with walls of a single layer of highly permeable endothelial cells. A human body has about 10 billion capillaries with a total surface area estimated to be 500 to 700 square meters (approximately the surface of a football field) (Guyton 1992).
2. Root hairs are microscopic extensions of root epidermal cells that greatly increase the surface area of the root, thus enabling greater capacity for absorption (Taiz & Zeigler 1991).
3. Kidney nephrons enable reabsorption of substances into the bloodstream. The concentrating ability of the mammalian kidney is closely related to the length of Henle's loop. Most mammalian kidneys have two types of nephrons. Some nephrons have long loops, while others have short ones. Animals that produce the most highly concentrated urine have only long-looped nephrons. The longer the loop the more reabsorption of water into the bloodstream (Schmidt-Neilsen 1997a).

C. Relationship between surface area and volume in indented objects.

Another method for increasing surface-to-volume ratio is for an object to be indented, or to have "bumps." The greater the number of indentions or bumps, the larger the increase in the surface-to-volume ratio, and subsequently the more efficient absorption of substances, filtering, and faster gas exchange.

Examples of this category include the following:

1. The presence of villi and microvilli in the small intestine of mammals increases the surface area of the small intestine hundreds of times, making this organ the main site of digestion and absorption (Thibodeau & Patton 1993b).
2. In fish, the gill surface area must be large enough to provide ade-

quate gas exchange. Highly active fish have the largest relative gill areas (Randall 1968).

D. Relationship between surface area and volume in objects that are sectioned into smaller units.

When large objects (large being a relative term) are sectioned into smaller units, the surface-to-volume ratio of that object increases. The more parts it is divided into, the greater the increase in the surface-to-volume ratio. This principle is found in many physiological processes, but perhaps the most obvious example is in the digestive system. Enzymes assist in the breakdown of food particles, but the smaller the particle the more efficient the enzymatic digestion. Most food compounds are either very large molecules (such as proteins), highly insoluble in water (such as fats) or large as well as insoluble (such as starch and cellulose). In order for the elements of the food to be absorbed and utilized they must be brought into soluble form and broken down into smaller component units (Schmidt-Nielson 1997b). Another example is the tiny alveolar sacs which comprise the human lungs. The division of the lung tissue into a large number of alveoli greatly increases the efficiency of gas exchange (Thibodeau & Patton 1993c).

Teaching Method & Conclusions

In order to demonstrate the above principles, we made four different groups of box shapes that we covered with graph paper. The boxes on the graph paper were one centimeter square. Figures 1 through 4 show the exact shapes used for teaching.

The structures in Figure 1 were used to demonstrate the changes that occur when bodies become flatter as described above, and correspond to Table 1. These boxes were prepared in the following sizes: $8 \times 8 \times 8$; $4 \times 8 \times 16$; $2 \times 16 \times 16$; $1 \times 32 \times 16$; $0.5 \times 32 \times 32$. (The paper used to make the shapes can be drawn once and then photocopied.)

The structures in Figure 2 were used to demonstrate the second principle described above concerning the changes that occur when an object is elongated, and correspond to Table 2. These boxes were prepared in the following sizes: $8 \times 8 \times 8$; $4 \times 8 \times 16$; $4 \times 4 \times 32$; $2 \times 4 \times 64$; $2 \times 2 \times 128$.

The structures in Figure 3 were used to demonstrate the third principle described above, that of the changes occurring when an object becomes indented. These boxes were prepared by gradually increasing the number of indentions in the original box shape.

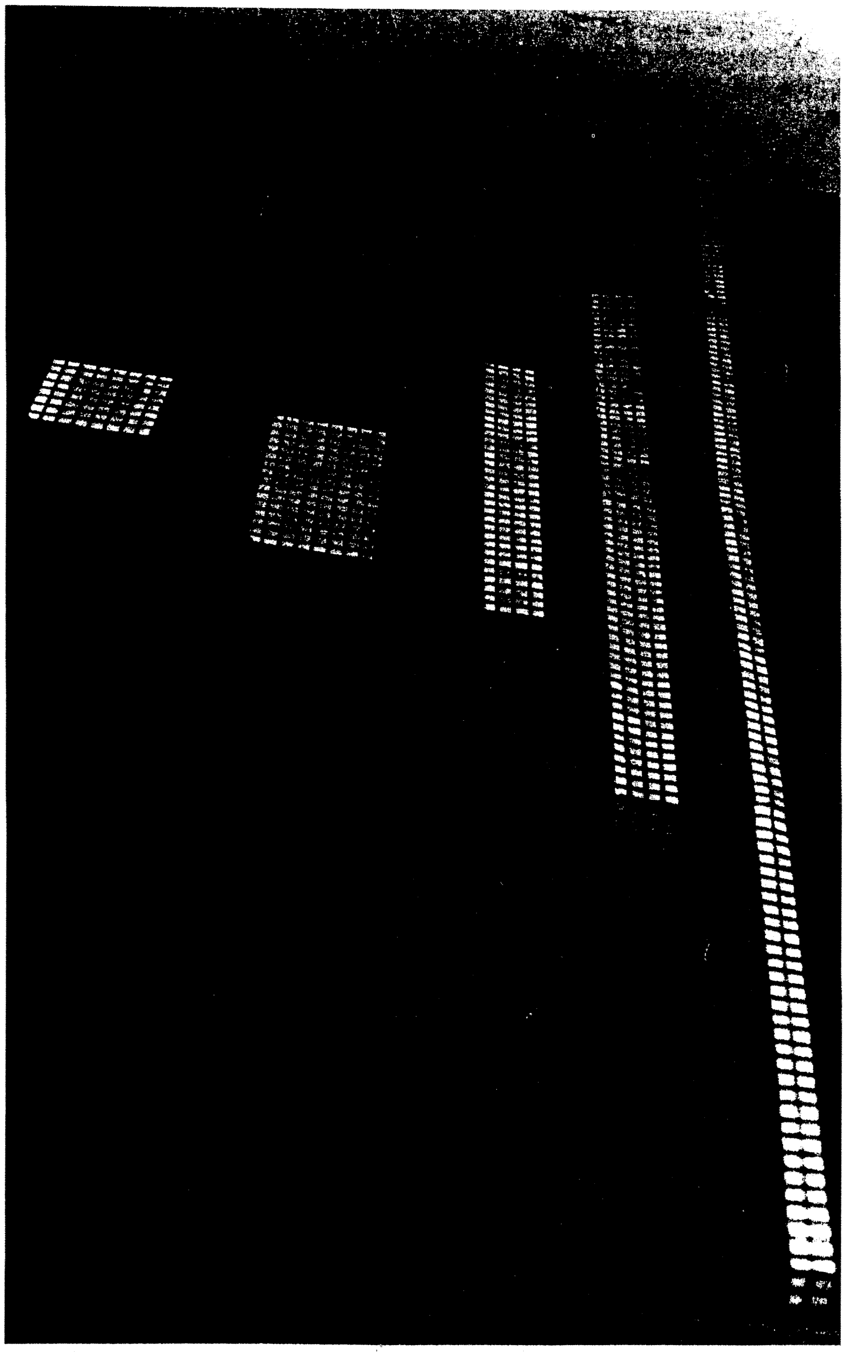

Figure 2. Models used to demonstrate the relationship between surface area and volume in elongated objects. The models go from a square box to more and more elongated shapes. (Each grid square represents one square cm.)

The structures in Figure 4 were used to demonstrate the fourth principle described above of the relationship between surface area and volume in objects that are sectioned into smaller units. The boxes prepared were as follows: one box $8 \times 8 \times 8$; 8 boxes $4 \times 4 \times 4$; 64 boxes $2 \times 2 \times 2$. These structures can be prepared together with the students, as we did.

After the structures were prepared, we asked the students to calculate the ratio between surface area and volume in the various systems, and to conclude how this ratio changes. In order to be certain that the concept was fully understood, we gave the class a quiz consisting of multiple choice questions demonstrating all of the above described principles (Ben Moreh 1992).

In conclusion, with the aid of these models we were able to take a relatively abstract biological concept and make it easier for our students to understand the concept of *the ratio between surface area and volume.*

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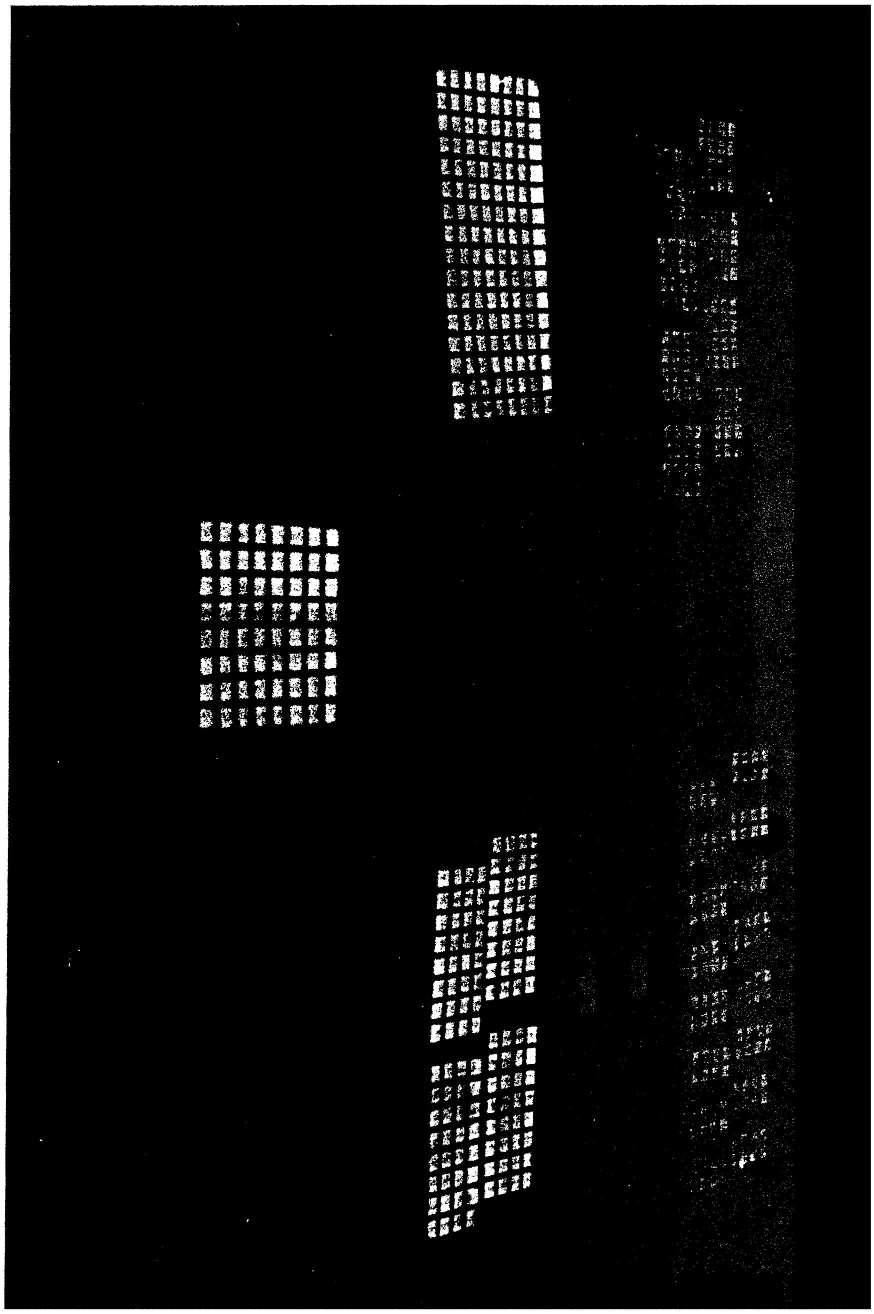

Figure 3. Models used to demonstrate the relationship between surface area and volume in segmented objects. The models go from a square box to more and more segmented shapes. (Each grid square represents one square cm.)

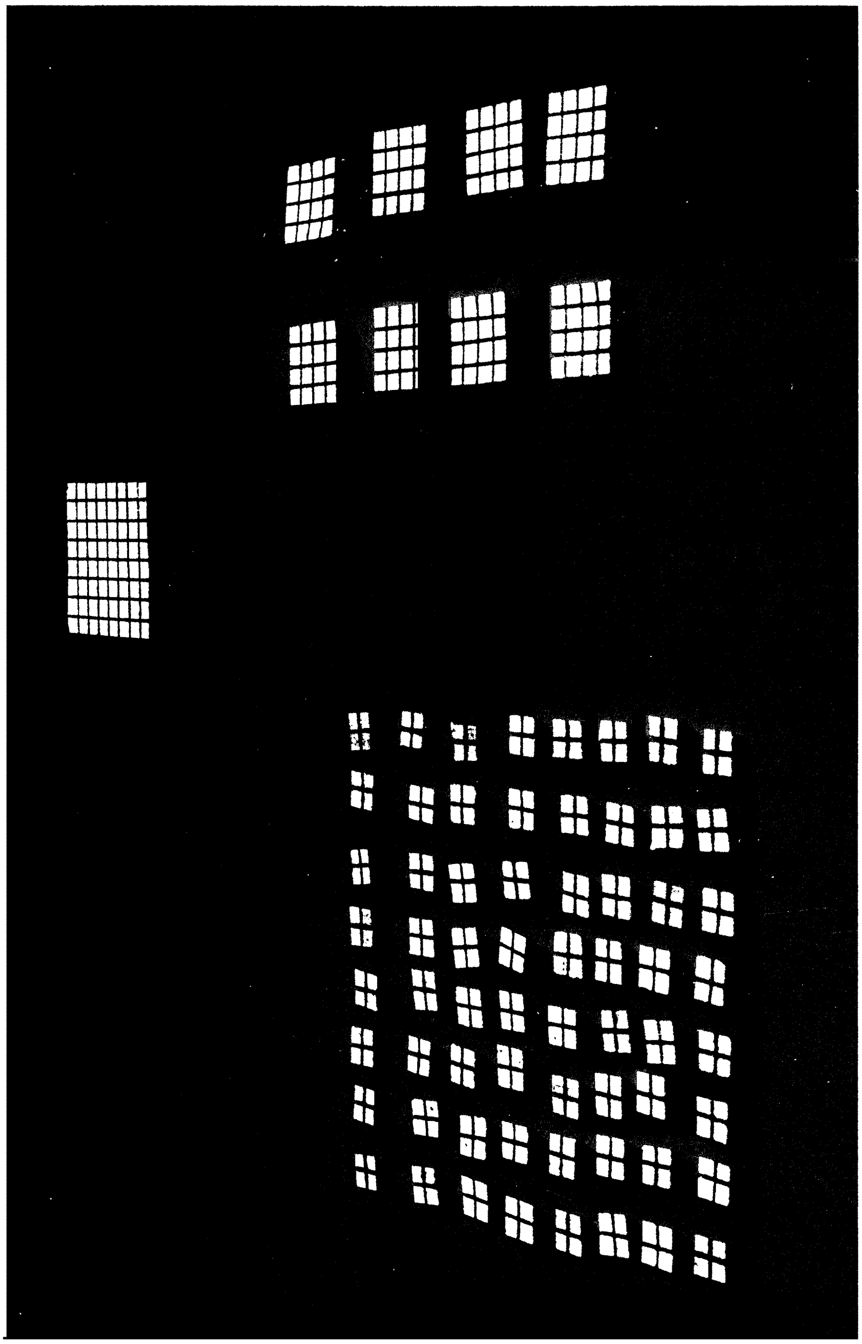

Figure 4. Models used to demonstrate the relationship between surface area and volume in sectioned objects. The models go from a square box to 8 smaller boxes to 64 even smaller boxes. (Each grid square represents one square cm.)